Efficient kriging for real-time spatio-temporal interpolation

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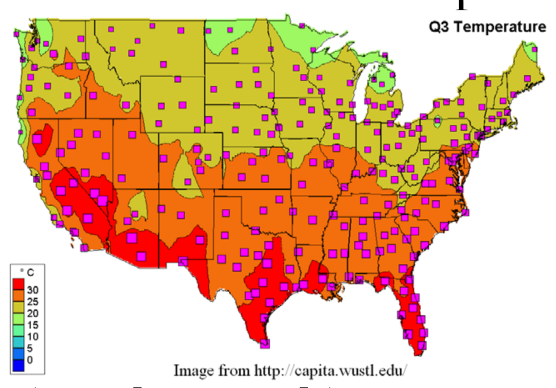
Kriging

- In atmospheric science, data is often collected at scattered station locations.
- Analysis require the data to be regularly-gridded or available at any query location of interest
  - Need a scheme to interpolate scattered station data.



- ***Kriging*** *(Isaaks et al.)* is a group of geostatistical techniques to interpolate the value of a random field (e.g., the temperature as a function of the geographic location) at an unobserved location from observations of its value at nearby locations.
- *Types:* simple, ordinary, universal, indicator, disjunctive, IRFk, lognormal, etc.
- **Ordinary kriging**➔Best Linear Unbiased Estimator

Ordinary kriging - formulation

- Given: Observed recordings $\{v_1,v_2,..,v_N\}$ at locations $\{x_1,x_2,..,x_N\}$
- Estimate $\{v_1^*,v_2^*,...,v_M^*\}$ at $\{x_1^*,x_2^*,..,x_M^*\}$
- Linear: $v_j^*=\Sigma w_{ij} v_i$
- Unbiased: $E(v_j^*)=E(v_i)$➔ $\Sigma_i w_{ij}=1$, for all j
- Best ➔ least variance

$$C_{i,k*} = \sum_{j=1}^{N} w_j C_{ij} + \mu$$

- C_{ij}=covariance between x_i and x_j
- Given by functional representation
  - Commonly used: Gaussian
  - $C_{ij}=exp(-||x_i-x_j||^2/h^2)+\delta(i,j)$
- μ ➔ *Lagrange multiplier*

Ordinary kriging - system

- For each $k^*=1,2,..,M$, solve

$$\begin{pmatrix} C_{11} & \dots & C_{1n} & 1\\ \vdots & \ddots & \vdots & \vdots\\ C_{n1} & \dots & C_{nn} & 1\\ 1 & \dots & 1 & 0\end{pmatrix} \times \begin{pmatrix} w_1\\ \vdots\\ w_N\\ \mu\end{pmatrix} = \begin{pmatrix} C_{1k*}\\ \vdots\\ C_{Nk*}\\ 1\end{pmatrix}$$

Kriging computational cost

- $O(N^4)$ complexity

$$\begin{pmatrix} v_{1*}\\ \vdots\\ v_{M*}\end{pmatrix} = \begin{pmatrix} v_1\\ \vdots\\ v_N\\ 0\end{pmatrix}^T \begin{pmatrix} C_{11} & \dots & C_{1N} & 1\\ \vdots & \ddots & \vdots & \vdots\\ C_{N1} & \dots & C_{NN} & 1\\ 1 & \dots & 1 & 0\end{pmatrix}^{-1} \begin{pmatrix} C_{1*1} & \dots & C_{1*N}\\ \vdots & \ddots & \vdots\\ \vdots & & \\ C_{M*1} & \dots & C_{M*N}\\ 1 & \dots & 1\end{pmatrix}$$

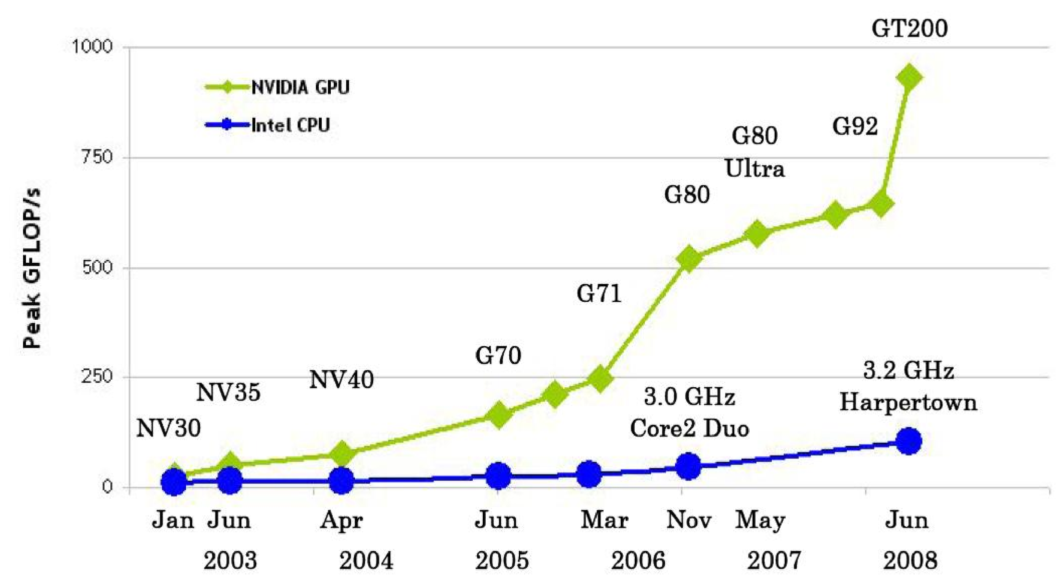
- $O(N^3)$ complexity

$$\begin{pmatrix} v_{1*}\\ \vdots\\ v_{M*}\end{pmatrix} = \begin{pmatrix} C_{1*1} & \dots & C_{1*N} & 1\\ \vdots & \ddots & \vdots & \vdots\\ \vdots & & & \\ C_{M*1} & \dots & C_{M*N} & 1\end{pmatrix} \begin{pmatrix} C_{11} & \dots & C_{1N} & 1\\ \vdots & \ddots & \vdots & \vdots\\ C_{N1} & \dots & C_{NN} & 1\\ 1 & \dots & 1 & 0\end{pmatrix}^{-1} \begin{pmatrix} v_1\\ \vdots\\ v_N\\ 0\end{pmatrix}$$

- Iterative solvers ➔ $O(kN^2)$
  - Saddle-point problem
  - Used SymmLQ *(Paige et al.)*

Graphical processing units (GPU)

- Graphics processors were developed to cater to the demands of real-time high definition graphics.
- Graphics processing units (GPU) are highly parallel, *multi-core processor* with *tremendous computational horsepower* and *high memory bandwidth.*



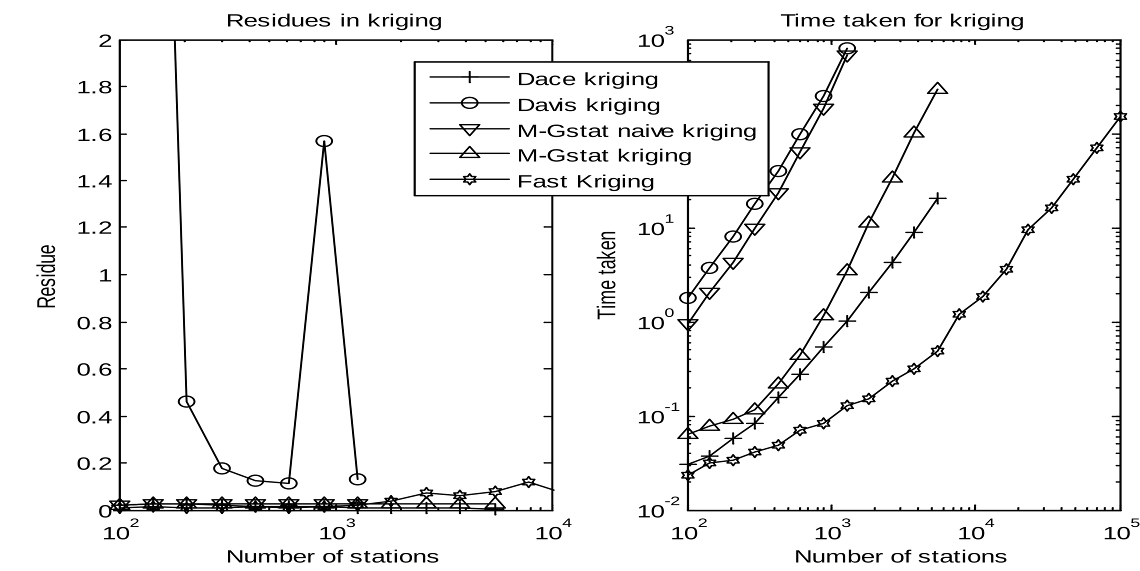
GT200 = GeForce GTX 280	G71 = GeForce 7900 GTX	NV35 = GeForce FX 5950 Ultra
G92 = GeForce 9800 GTX	G70 = GeForce 7800 GTX	NV30 = GeForce FX 5800
G80 = GeForce 8800 GTX	NV40 = GeForce 6800 Ultra	

GPU-accelerated kriging

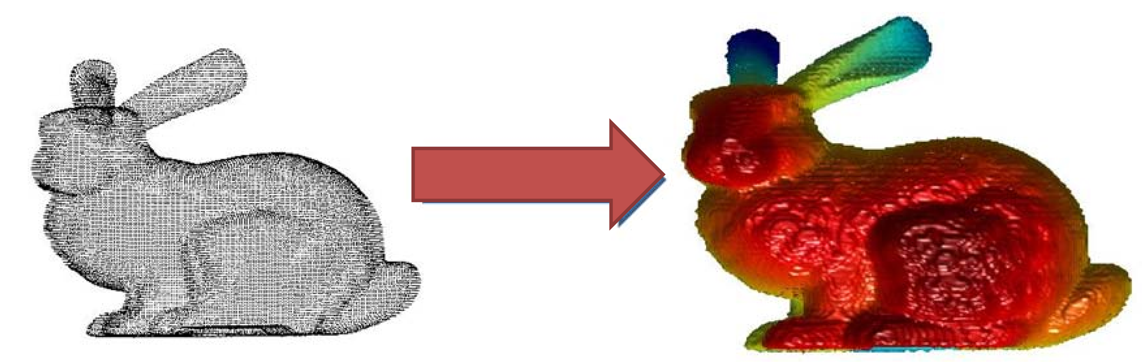
- Used the open source **GPUML** *(Srinivasan et al.)*
  - Speedup: ~400X over a direct implementation.

Performance comparison

- Fastest among many available kriging and fitting tools



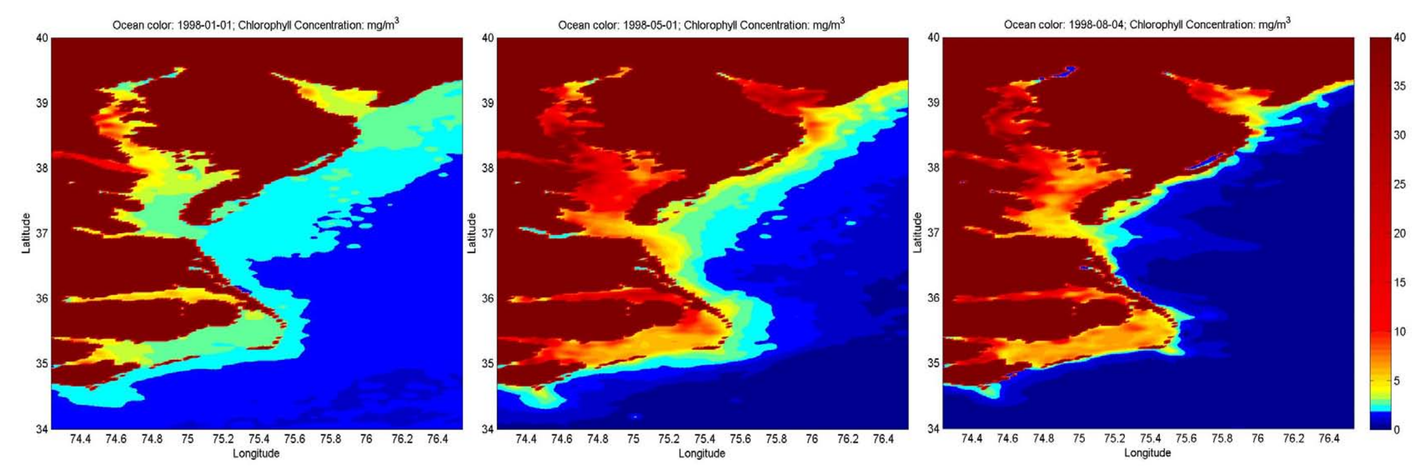
Spatial example



- Kriged at *8 million points* using *100,000 points* in *7minutes*, a direct implementation would take almost 2 days

Chesapeake bay ocean color

- Spatio-temporal kriging
- Kriged using *288 x 240 7*-day gridded data
- ~ *70-80s* to krige for *1day* prediction (direct implementaion would take approximately *8 hours*)



References

- *Isaaks, E and Srivastava R*, "**Applied Geostatistics**", Oxford University Press, New York, 542 pp, 1989.
- *Paige, CC and Saunders, MA*, "**Solution of sparse indefinite systems of linear equations**", SINUM 12, 617--629, 1975.
- *Srinivasan BV and Duraiswami R*, "**Scaling kernel machine learning algorithm via the use of GPUs**", GPU Technology Conference, NVIDIA Research Summit, 2009.

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PIRL Perceptual Interfaces and Reality Laboratory